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SIMPLIFIED POINT AND INTERVAL ESTIMATION FOR REMOVAL TRAPPING.(U)
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MRC Technical Summary Report #1704

SIMPLIFIED POINT AND INTERVAL ESTIMATION FOR REMOVAL TRAPPING

Andrew P. Soms

Mathematics Research Center
University of Wisconsin–Madison
610 Walnut Street
Madison, Wisconsin 53706

December 1976

(Received November 12, 1976)

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UNIVERSITY OF WISCONSIN - MADISON
MATHEMATICS RESEARCH CENTER

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ABSTRACT

Two methods, the generalized moment and the regression, both based on the limiting distribution of the multinomial, are given for estimating the parameters in the removal trapping method of estimating animal and insect populations. Some finite sample size results are provided indicating the speed of convergence to the limiting distribution. Numerical examples are also discussed.

AMS(MOS) Subject Classification - 62E20, 62F25, 62P10

Key Words - Removal trapping, moment estimator, regression estimator, asymptotic confidence intervals

Work Unit Number 4 - Probability, Statistics, and Combinatorics

† University of Wisconsin-Milwaukee, Milwaukee, Wisconsin 53201. The author would like to thank Joan Jass of the Milwaukee Public Museum for pointing out this problem.

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SIMPLIFIED POINT AND INTERVAL ESTIMATION
FOR REMOVAL TRAPPING

Andrew P. Soms†

1. Introduction

A thorough discussion of the removal trapping method of estimating animal and insect populations, together with limitations, is given in [6], pp. 182-6. It is pointed out in [4] that this method is particularly suited for insect populations. Briefly, there are assumed to be m organisms in some fixed area, k trapping or sweeping periods, $k \geq 2$, and each organism is assumed to have a constant probability p of being captured in any of the k periods, independent of the other organisms (The organisms are not released when captured). If the trapping probability is p, then, as pointed out by Moran [5], p. 308, the joint density of the n_i, $1 \leq i \leq k$, the number of organisms trapped in each of the periods, is

$$p[n_i = s_i, \sum_1^k s_i \leq m] = \frac{m!}{s_1! \ldots s_k!(m-\sum_1^k s_i)!} p^{s_1}(p(1-p))^{s_2} \ldots (p(1-p)^{k-1})^{s_k} \cdot$$

$$\cdot ((1-p)^k)^{m-\sum_1^k s_i} \frac{m!}{s_1! \ldots s_k!(m-\sum_1^k s_i)!} p^{\sum_1^k s_i} (1-p)^{\sum_2^k (i-1)s_i + k(m-\sum_1^k s_i)},$$

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where $0 < p < 1$. The above is seen to be a multinomial distribution, with $k+1$ categories, and parameters m and $p_i = p(1-p)^{i-1}$, $1 \leq i \leq k$, and $p_{k+1} = (1-p)^k = 1 - \sum_1^k p_i = 1 - \sum_1^k p(1-p)^{i-1}$. It is desired to estimate m and p and give asymptotically exact confidence intervals. In [5] a method based on maximum likelihood is proposed, which is elaborated upon by Zippin [7]. In addition to replacing $m!$ by Stirling's approximation, the effect of which is not clear, both Moran [5] and Zippin [7] state that the usual regularity conditions for the joint asymptotic normality of the maximum likelihood estimators are not satisfied in this case (in addition to other assumptions, it is assumed that the parameters being estimated, m and p, remain constant, which is not true here, since the asymptotic behavior is for fixed p as $m \to \infty$), and then they proceed in the hope that somehow a justification may be produced without giving it. Further, even if these difficulties are neglected, the estimating equations are either implicit, requiring iteration, or after some approximations, require charts. Here two theoretically justifiable methods are discussed, both based on the limiting distribution of the multinomial, which give the estimates explicitly as functions of $n_1, \ldots, n_k$. The first is a modified method of moments and the second is based on regression estimates.

2. The Modified Moment Estimates

Since $En_i/En_{i-1} = 1 - p = q$, $2 \le i \le k$, equating expectations to the observed values gives $n_i/n_{i-1} = 1 - \hat{p} = \hat{q}$. Note that this is defined only if $n_{i-1} > n_i$. To minimize this effect it is reasonable to take the geometric mean of the $k - 1$ estimates to obtain the estimate

$$\hat{q} = 1 - \hat{p} = \left(\frac{n_2}{n_1}\,\frac{n_3}{n_2}\cdots\frac{n_k}{n_{k-1}}\right)^{1/(k-1)} = \left(\frac{n_k}{n_1}\right)^{1/(k-1)},$$

and this is the estimate to be considered here – note that it fails to exist only if $n_k \ge n_1$, an event which will be shown to have limiting probability 0. Since $E(\sum_1^k n_i) = m(1-(1-p)^k)$, the moment estimate $\hat{m}$ of m is

$$\hat{m} = \frac{\sum_1^k n_i}{1-(1-\hat{p})^k}.$$

Consider now the problem of asymptotically exact confidence intervals – a reasonable assumption is that p stays constant and $m \to \infty$. The asymptotic distributions of $\hat{p}$ and $\hat{m}$ will be obtained by using two results – the joint asymptotic normality of $\tilde{n} = (n_1, \ldots, n_k)$ and a result given in Anderson [1], pp. 76-7. It is well known that as $(p_1, \ldots, p_k)$ stays constant and $m \to \infty$,

$$\left(\frac{n_i - mp_i}{(mp_iq_i)^{\frac{1}{2}}},\ 1 \le i \le k\right) \xrightarrow{w} N(\tilde{0}, R), \tag{2.1}$$

("$\xrightarrow{w}$" means convergence in distribution), where

$$R = [\rho_{ij}], \quad \rho_{ii} = 1 \text{ and for } i \neq j, \quad \rho_{ij} = -(p_i p_j/(q_i q_j))^{\frac{1}{2}}$$

(see, e.g. Johnson and Kotz, [3], p. 284) – recall that here $p_i = p(1-p)^{i-1}$, $1 \leq i \leq k$ (also for notational convenience $p_1 = p, q_1 = 1 - p_1 = q$), and hence it suffices to keep p constant. The result cited in Anderson is: Let $f(\tilde{x})$ be a function with first and second derivatives existing in a neighborhood of $\tilde{x} = \tilde{b}$, $\tilde{b} = (b_1, \ldots, b_k)$ a fixed vector, and suppose $\sqrt{n}(U(n) - \tilde{b}) \xrightarrow{w} N(\tilde{0}, T)$. Then

$$\sqrt{n}(f(U(n)) - f(b)) \xrightarrow{w} N(0, \Phi_b' T \Phi_b) , \tag{2.2}$$

where $\Phi_b' = (\frac{\partial f}{\partial x_1}, \ldots, \frac{\partial f}{\partial x_k})_{\tilde{b}}$. The asymptotic distribution of $1 - \hat{p}$ will now be obtained. It follows from (2.1) that

$$\sqrt{m}(n_i/(m(p_i q_i)^{\frac{1}{2}}) - (p_i/q_i)^{\frac{1}{2}}, \quad 1 \leq i \leq k) \xrightarrow{w} N(0, R) , \tag{2.3}$$

where $R = [\rho_{ij}]$, $\rho_{ii} = 1$, and for $i \neq j$, $\rho_{ij} = -(p_i p_j/(q_i q_j))^{\frac{1}{2}}$. Take $f(\tilde{x})$ to be

$$f(\tilde{x}) = \left(\frac{x_k \sqrt{p_k q_k}}{x_1 \sqrt{p_1 q_1}} \right)^{1/(k-1)} . \tag{2.4}$$

In all that follows, $\tilde{b} = ((p_1/q_1)^{\frac{1}{2}}, \ldots, (p_k/q_k)^{\frac{1}{2}})$. Then, using (2.2),

$$\sqrt{m}((n_k/n_1)^{1/(k-1)} - f(\tilde{b})) = \sqrt{m}((n_k/n_1)^{1/(k-1)} - (1-p)) \xrightarrow{w}$$

$$N(0, (\frac{1-p}{k-1})^2 (\frac{q}{p} + \frac{1-pq^{k-1}}{pq^{k-1}} + 2) = N(0, \sigma_p^2) ,$$

since

$$\left(\frac{\partial f}{\partial x_1}\right)_{\tilde{b}} = -(1/(k-1))\, q(q/p)^{\frac{1}{2}} ,$$

$$\left(\frac{\partial f}{\partial x_k}\right)_{\tilde{b}} = (1/(k-1))\, q((1-pq^{k-1})/(pq^{k-1}))^{\frac{1}{2}} , \tag{2.5}$$

and $\Phi_b' R\Phi_b = \sigma_p^2$. Therefore also

$$\sqrt{m}\,(\hat{p}-p) \xrightarrow{w} N(0,\sigma_p^2) ,$$

where $\hat{p} = 1-(n_k/n_1)^{1/(k-1)}$, or, equivalently, $\hat{p}$ is asymptotically $N(p,\sigma_p^2/m)$. Using the same technique on $\hat{m}$, let

$$g(\tilde{x}) = \left(\sum_1^k x_i(p_iq_i)^{\frac{1}{2}}\right) \Bigg/ \left(1-\left(\frac{x_k\sqrt{p_kq_k}}{x_1\sqrt{p_1q_1}}\right)^{k/(k-1)}\right). \tag{2.6}$$

Then

$$\left(\frac{\partial g}{\partial x_i}\right)_{\tilde{b}} = (p_iq_i)^{\frac{1}{2}}/(1-q^k) \quad \text{for } 2 \le i \le k-1 ,$$

$$\left(\frac{\partial g}{\partial x_1}\right)_{\tilde{b}} = ((pq)^{\frac{1}{2}} - (k/(k-1))q^k(q/p)^{\frac{1}{2}})/(1-q^k) , \tag{2.7}$$

$$\left(\frac{\partial g}{\partial x_k}\right)_{\tilde{b}} = ((p_kq_k)^{\frac{1}{2}} + (k/(k-1))q(p_kq_k)^{\frac{1}{2}}/p)/(1-q^k) .$$

Since $g(\tilde{b}) = 1$, by (2.2)

$$\sqrt{m}\,(\hat{m}/m-1) \xrightarrow{w} N(0, \Phi_b' T\Phi_b) ,$$

where $\Phi_b' = \left(\frac{\partial g}{\partial x_1}, \ldots, \frac{\partial g}{\partial x_k}\right)_{\tilde{b}}$. In order to evaluate $\Phi_{\tilde{b}}' T\Phi_b$, let $a' = ((p_1q_1)^{\frac{1}{2}}, \ldots, (p_kq_k)^{\frac{1}{2}})$,

and $b' = (-(k/(k-1))(1-p)^k (q/p)^{\frac{1}{2}}, 0, \ldots, 0, (k/(k-1))(1-p)(p_k q_k)^{\frac{1}{2}}/p)$. Then $\Phi_b' T\Phi_b = a'Ta + 2a'Tb + bTb$, and after some algebra,

$$b'Tb = (k/(k-1))^2 (q^{k+1}/p)(q^k + pq^{k-1} + 1) \ ,$$

$$a'Tb = 0 \ ,$$

$$a'Ta = 1 - q^k - (1-q^k)^2 \ .$$

Hence $\Phi_b' T\Phi_b = ((k/(k-1))^2 (q^{k+1}/p)(1+pq^{k-1}+q^k) + q^k(1-q^k))/(1-q^k)^2 = \sigma_m^2$.

So $\sqrt{m}(p-\hat{p}) \xrightarrow{w} N(0, \sigma_p^2)$ and $(\hat{m}-m)/\sqrt{m} \xrightarrow{w} N(0, \sigma_m^2)$. Since $\hat{p} \to p$ and $\hat{m}/m \to 1$ in probability, $\sigma_{\hat{p}}^2 \to \sigma_p^2$, $\sigma_{\hat{m}}^2 \to \sigma_m^2$ in probability ($\sigma_{\hat{p}}^2$ and $\sigma_{\hat{m}}^2$ are obtained from σ_p^2 and σ_m^2 by replacing p by $\hat{p}$, q by $\hat{q}$) and the limiting distribution of both $(\hat{p}-p)/(\sigma_{\hat{p}}/\hat{m}^{\frac{1}{2}})$ and $(\hat{m}-m)/(\sigma_{\hat{m}}\hat{m}^{\frac{1}{2}})$ is the standardized normal, and therefore asymptotically exact (marginal) $1-\alpha$ confidence intervals for p and m are

$$\hat{p} \pm z_\alpha \sigma_{\hat{p}}/\hat{m}^{\frac{1}{2}} \ ,$$

$$\hat{m} \pm z_\alpha \sigma_{\hat{m}} \hat{m}^{\frac{1}{2}} \ ,$$

where z_α is the upper $100\alpha^{\underline{th}}$ percentile of the standardized normal. A Monte Carlo example is now given to illustrate the asymptotic theory. Three cases were considered: $p = .4$, $k = 3$, and $m = 100$, 200, and 400. In each case 1000 random samples (using computer generated random numbers) were taken and for each sample the point estimates and confidence intervals calculated. Two coverage relative frequencies were computed – the relative frequency C1 of the estimate lying within $\pm\sigma$ and $\pm 2\sigma$ of the true value, and the relative frequency C2 of the estimated confidence interval covering the true parameter value – the latter is, of course, of

the greatest interest. The results are given in Table 1. Here σ is the theoretical standard deviation (st. dev.) and $\hat{\sigma}$ is the standard deviation estimated from a sample.

1. Monte Carlo Results for 1000 Samples

a. p

p	m	Sample Mean	σ	Sample st. dev.	$C1(\pm 1\sigma)$	$C1(\pm 2\sigma)$	$C2(\pm 1\hat{\sigma})$	$C2(\pm 2\hat{\sigma})$
.4	100	.397	.092	.090	.68	.96	.70	.97
.4	200	.399	.065	~~.695~~ .065	.68	.96	.68	.96
.4	400	.401	.046	.045	.70	.96	.70	.96

b. m

p	m	Sample Mean	σ	Sample st. dev.	$C1(\pm 1\sigma)$	$C1(\pm 2\sigma)$	$C2(\pm 1\hat{\sigma})$	$C2(\pm 2\hat{\sigma})$
.4	100	105.0	13.7	22.1	.69	.92	.77	.92
.4	200	204.1	19.4	22.3	.69	.93	.73	.93
.4	400	403.7	27.5	28.8	.69	.94	.69	.94

An alternative procedure to using $\hat{\sigma}$ in the confidence intervals is to correct for bias in the point estimate (e.g., for $m = 100$, 5.0 is subtracted from the estimate and this is considered to be the new estimate) and to use the sample standard deviation s (here based on 1000 random samples). As might be expected, this results in slightly conservative intervals. The results are given in Table 2, based on 1000 simulations in each case. In practice the sample estimates would be used as the true values in the simulation.

2. Coverages with Bias Substracted
when Sample Standard Deviation is Used

a. p

p	m	C1(±s)	C1(±2s)	C2(±s)	C2(±2s)
.4	100	.68	.96	.70	.97
.4	200	.68	.96	.68	.96
.4	400	.70	.96	.70	.96

b. m

p	m	C1(±s)	C1(±2s)	C2(±s)	C2(±2s)
.4	100	.61	.93	.89	.97
.4	200	.65	.94	.73	.96
.4	400	.67	.95	.69	.96

Sometimes before the trapping experiment is begun, a preliminary estimate of p is available. In this case k, the number of trapping periods, can be chosen so as to minimize the variance of p – because of the importance of p in this method of estimation this is a reasonable optimality criterion. The function to be minimized, apart from multiplicative constants, is $f(k) = 1/(k-1)^2(1+1/q^{k-1})$. The minimum is obtained by setting the derivative equal to 0, the resultant equation being

$$q^z + 1 = (-\ell n\, q/2)z \ ,$$

$z = k - 1$. Table 3 gives the nearest integer to the exact minimum value, which is readily obtained by iteration, as a function of selected q .

3. Optimum k Values

q	k
.95	44
.90	22
.85	15
.80	11
.70	7
.60	5
.50	4

The customary statistic used to test the adequacy of the model is $Z = \sum_1^k (n_i - \hat{m}\hat{p}_i)^2/(\hat{m}\hat{p}_i)$, where $\hat{p}_i = \hat{p}(1-\hat{p})^{i-1}$. It is not at all clear, in this or, of course, the maximum likelihood case, that Z has an asymptotic (p fixed, $m \to \infty$) χ^2 distribution *with* $k-2$ degrees of freedom (d. f.), since the usual regularity conditions (see [2], pp. 500-1, 506) are not satisfied. The empirical approach given here consists of using (2.2) to obtain the expected value of the limiting distribution of Z and then to fit a χ^2 distribution (as is done with good results in fitting the distribution of sums of χ^2 random variables) by estimating the d. f. using the parameter estimates. The observed value of Z is then compared to the upper $100\alpha^{\underline{th}}$ percentile of the fitted χ^2 (using interpolation on the d. f., since in general the fitted d. f. will not be integral). Specifically, consider

$$f_i(\tilde{x}) = x_i(p_i q_i)^{\frac{1}{2}} - g(\tilde{x})(1 - f(\tilde{x}))(f(\tilde{x}))^{i-1} \quad ,$$

where $f(\tilde{x})$ and $g(\tilde{x})$ are given by (2.4) and (2.6), respectively. Note that $f_i(\tilde{b}) = 0$ and thus from (2.2),

$$\sqrt{m} f_i(n_1/(m(p_1 q_1)^{\frac{1}{2}}), \ldots, n_k/(m(p_k q_k)^{\frac{1}{2}})) = \sqrt{m} f_i(\tilde{u})$$

$$= \sqrt{m}((n_i - \hat{m}\hat{p}\hat{q}^{i-1})/m) \xrightarrow{w} N(0, \sigma_i^2) \ , \qquad (2.8)$$

where $\sigma_i^2 = \Phi'_{i\tilde{b}} T \Phi_{i\tilde{b}}$, $\Phi'_{i\tilde{b}} = (\frac{\partial f_i(\tilde{x})}{\partial x_1}, \ldots, \frac{\partial f_i(\tilde{x})}{\partial x_k})_{\tilde{b}}$.

Using (2.5) and (2.7) and the chain rule, an expression for σ_i^2 can be obtained, e.g.,

$$\left(\frac{\partial f_i(\tilde{x}_i)}{\partial x_i}\right)_{\tilde{b}} = (p_i q_i)^{\frac{1}{2}} - \left(\frac{\partial g(\tilde{x})}{\partial x_i}\right)_{\tilde{b}} pq^{i-1}, \quad 2 \leq i \leq k-1 \ ,$$

and similarly for the other derivatives. Even though explicit expressions do not appear practical, σ_i^2 is easily evaluated by means of a short computer program. Since $(\hat{m}/m)^{\frac{1}{2}} \to 1$ and $\hat{p}_i \to p$ in probability, it follows from (2.8) that

$$(n_i - \hat{m}\hat{p}\hat{q}^{i-1})/(\hat{m}\hat{p}_i)^{\frac{1}{2}} \to N(0, \sigma_i^2/p_i) \ ,$$

and therefore the asymptotic mean of $Z = \sum_1^k (n_i - \hat{m}\hat{p}\hat{q}^{i-1})^2/(\hat{m}\hat{p}_i)$ is $\mu = \sum_1^k \sigma_i^2/p_i$. Replacing the parameters p_i and q_i by their estimates $\hat{p}_i$, $\hat{q}_i$ in μ gives the estimated d.f. of the distribution of Z, and using these d.f. a cut-off point for the adequacy of fit test can be obtained from tables.

3. Regression Estimates

The method of moments given above depends on the "extreme" values n_1 and n_k and therefore in cases where the trapping periods are too short and hence the data show considerable fluctuation, it may be preferable to use an estimate of p depending on all the data. As suggested in [4], a simple check is to plot n_i against i on semi-log paper and see whether the plot approximates a straight line. If yes, the method of moments with its attendant computational simplicity can be used, and if no, then the regression method discussed below may be used.

It is pointed out in [5] that $\log En_i = \log \mu_i$ lie on the straight line

$$\log \mu_i = i \log(1-p) - \log(1-p) + \log p + \log m \quad , \qquad (3.1)$$

but this method then is dismissed in [5] by saying that the usual assumptions of regression theory are not satisfied. Here a different approach is taken – namely, the point estimator of p suggested by regression theory is used but then, in place of the usual regression theory, the limiting distribution is obtained from (2.2). The regression equation suggested by (3.1) is

$$\log n_i = i\beta + \alpha + \varepsilon_i, \quad 1 \leq i \leq k \quad ,$$

where $\beta = \log(1-p)$ (to any base), α a constant, and ε_i the error term which will be of no interest here. The least squares estimate $\hat{\beta}$ of β is

$$\hat{\beta} = \sum_1^k (\log n_i)(i - \frac{k+1}{2}) / \sum_1^k (i - \frac{k+1}{2})^2 \quad .$$

Since $\sum_{1}^{k} i^2 = (k)(k+1)(2k+1)/6$,

$$\hat{\beta} = \sum_{1}^{k} (\log n_i)(i - \frac{k+1}{2})/(k(k^2-1)/12) \ .$$

Then the corresponding estimator $1-\hat{p}$ of $1-p$ is

$$1-\hat{p} = \prod_{1}^{k} n_i^{c_i} \ , \qquad (3.2)$$

where $c_i = (i-(k+1)/2)/(k(k^2-1)/12)$. In order to show that (3.2) is consistent for p and to obtain its asymptotic variance, it is just as easy to consider general estimates of $1-p$ of the form (3.2) with the c_i arbitrary and to determine the conditions on c_i needed for consistency. Let

$$h(\tilde{x}) = \prod_{1}^{k} (x_i(p_i q_i)^{\frac{1}{2}})^{c_i} \ . \qquad (3.3)$$

Then, using (2.2) and $p_i = p(1-p)^{i-1}$,

$$\sqrt{m}\left(\prod_{1}^{k} n_i^{c_i}/m^{\sum_{1}^{k} c_i} - \prod_{1}^{k} p_i^{c_i}\right)$$

$$= \sqrt{m}\left(\prod_{1}^{k} n_i^{c_i}/m^{\sum_{1}^{k} c_i} - p^{\sum_{1}^{k} c_i}(1-p)^{\sum_{1}^{k-1} ic_{i+1}}\right) \ .$$

Therefore a sufficient condition for consistency is $\sum_{1}^{k} c_i = 0$ and $\sum_{1}^{k-1} ic_{i+1} = 1$, and in this case

$$\sqrt{m}\,\left(\prod_1^k n_i^{c_i} - (1-p)\right) \xrightarrow{w} N(0,\sigma_p^2) \quad ,$$

where σ_p^2 is determined in the usual way from (3.3) using (2.2). It is noted that for $c_i = (i-(k+1)/2)/(k(k_k^2-1)/12)$ these two conditions are satisfied, since clearly $\sum_1^k c_i = 0$ and, letting $c = k(k^2-1)/12$,

$$\sum_1^{k-1} ic_{i+1} = \sum_1^{k-1} i(i+1-\frac{k+1}{2})/c = (\sum_1^{k-1} i^2 - (k-1)/2 \sum_1^{k-1} i)/c$$

$$= ((k-1)(k)(2k-1)/6 - k(k^2-1)/4)/c = 1 \quad .$$

So, for this choice of c_i, $\sigma_p^2 = \Phi'_{\tilde{b}} T \Phi_{\tilde{b}}$,

$$\Phi'_b = (\frac{\partial h(\tilde{x})}{\partial x_1}, \ldots, \frac{\partial h(\tilde{x})}{\partial x_k})_{\tilde{b}} \text{ and since } \frac{\partial h(\tilde{x})}{\partial x_i} = (c_i/x_i)\prod_1^k (x_i(p_i q_i)^{\frac{1}{2}})^{c_i} \quad ,$$

$$(\frac{\partial h(\tilde{x})}{\partial x_i})_{\tilde{b}} = c_i(q_i/p_i)^{\frac{1}{2}} q \quad . \tag{3.4}$$

Therefore, after some algebra, $\sigma_p^2 = q^2(\sum_1^k c_i^2(q_i/p_i) - 2\sum\sum_{i<j} c_i c_j)$.

A satisfactory estimator of $\hat{m}$ is obtained by the same argument as for the moment estimator, namely,

$$\hat{m} = \sum_1^k n_i/(1-(1-\hat{p})^k) \quad ,$$

where $1-\hat{p}$ is now given by (3.2). As before, let

$$g(\tilde{x}) = \sum_1^k x_i\sqrt{p_i q_i}\,/(1-(\prod_1^k (x_i\sqrt{p_i q_i})^{c_i})^k) \quad .$$

Then, by (2.2),

$$\sqrt{m}\,(\frac{\hat{m}}{m} - 1) \xrightarrow{w} N(0, \sigma_m^2) \quad ,$$

where $\sigma_m^2 = \Phi_{\tilde{b}}' T \Phi_{\tilde{b}}$, $\Phi_{\tilde{b}}' = (\frac{\partial g}{\partial x_1}, \ldots, \frac{\partial g}{\partial x_k})_{\tilde{b}}$ and

$$(\frac{\partial g}{\partial x_i})_{\tilde{b}} = \frac{(p_i q_i)^{\frac{1}{2}} + k c_i (q_i/p_i)^{\frac{1}{2}} (1-p)^k}{1 - (1-p)^k} \quad . \qquad (3.5)$$

Therefore, after some algebra, $\sigma_m^2 = \frac{q^k}{1-q^k} + \frac{k^2 q^{2k}}{(1-q^k)^2} (\sum_1^k c_i^2 \frac{q_i}{p_i} - 2 \sum_{i<j} c_i c_j)$.

It should be noted that the moment estimator of $1 - p$, $(n_k/n_1)^{1/(k-1)}$, is a special use of (3.2) with $c_1 = -1/(k-1)$, $c_i = 0$, $2 \le i \le k-1$, and $c_k = 1/(k-1)$, since $c_1 + c_k = 0$ and $\sum_1^{k-1} i c_{i+1} = (k-1)/(k-1) = 1$.

The discussion of lack of fit is completely analogous to the method of moments, except that the partials are different but again obtained by the chain rule using (3.4) and (3.5) and evaluated by a short computer program. Specifically, let $f_i(\tilde{x}) = x_i (p_i q_i)^{\frac{1}{2}} - g(\tilde{x})(1-h(\tilde{x}))(h(\tilde{x}))^{i-1}$.

Then

$$(\frac{\partial f_i}{\partial x_j})_{\tilde{b}} = -(\frac{\partial g(\tilde{x})}{\partial x_j})_{\tilde{b}}\, p q^{i-1} - (i-1) p q^{i-2} (\frac{\partial h(\tilde{x})}{\partial x_j})_{\tilde{b}} + q^{i-1} (\frac{\partial h(\tilde{x})}{\partial x_j})_{\tilde{b}} \quad ,$$

for $i \neq j$, and if $i = j$, $(p_i q_i)^{\frac{1}{2}}$ is added to the above expression in which j has been replaced by i .

4. Numerical Examples

The two methods of estimation discussed above are applied to the data in [4] and [5]. First, consider the rat data in [5] consisting of $k = 18$, $\tilde{n} = (49, 32, 31, 34, 16, 33, 22, 27, 17, 19, 18, 16, 18, 12, 14, 12, 17, 7)$. If a semi-log plot of n_i against i is made, it can be seen that there is a large variability in the data, suggesting that the intervals are too short. In addition, n_1 appears large and n_{18} small compared to the line suggested by the other points, and thus the moment method should underestimate m, which Table 4 shows (This is also reflected in the large calculated Z value). If however the data are grouped into 9 trapping periods with $n_{gi} = n_{2i-1} + n_{2i}$, $i = 1, 2, \ldots, 9$, the semi-log plot is much smoother and the moment method gives comparable point estimates to the regression method, even though the variances are somewhat larger, as is seen in Table 4. The maximum likelihood estimates and their estimated standard deviations (s.d.'s) given in [5] are $\hat{m} = 520$, 32.9 and $\hat{p} = .0756$, $.00933$ (the estimated standard deviations for the estimates discussed here are understood to be $\sigma_{\hat{p}} / \hat{m}^{\frac{1}{2}}$ for $\hat{p}$ and $\sigma_{\hat{m}} \hat{m}^{\frac{1}{2}}$ for $\hat{m}$). Note that in terms of p for the original data, the probability of capture p' for the grouped data is $p' = p + p(1-p)$.

4. Moment and Regression Estimates for the Rat Data

Parameter	Original data		Grouped data	
	Moments	Regression	Moments	Regression
p	.108	.0737	.141	.139
s.d.	.0212	.0100	.0260	.0186
m	452	527	529	533
s.d.	29.4	37.0	51.0	39.0
		Test of fit		
Z	31.9	17.4	2.00	2.08
d.f.	23.3	16.1	11.2	7.11

It is pointed out in [4] that removal trapping is sometimes the only feasible method of estimating insect populations. The example discussed there consists of the number of maccolaspis flavida, n_i, caught in $k = 10$ sweeping periods (the data in [4] were actually grouped), with $\tilde{n} = (72, 63, 44, 32, 31, 23, 17, 18, 11, 13)$. Using a somewhat involved graphical method, which does not yield any interval estimates, Menhinick obtains $\hat{p} = .212$ and $\hat{m} = 359$. The complete results for this example, for both the moment and regression method, are given in Table 5.

5. Moment and Regression Estimates and Fitted Values for the Insect Data

Parameter	Moments	Regression
p	.173	.188
s.d.	.0289	.0198
m	381	370
s.d.	24.8	14.8
	Test of fit	
Z	4.59	3.28
d.f.	12.3	8.29

Predicted Values $\hat{n}_i$

Data	$\hat{n}_i$	$\hat{n}_i - n_i$	$\hat{n}_i$	$\hat{n}_i - n_i$
72	66.0	-6.0	69.5	-2.5
63	54.5	-8.5	56.5	-6.5
44	45.1	1.1	45.9	1.9
32	37.3	5.3	37.3	5.3
31	30.8	-.2	30.3	- .7
23	25.5	2.5	24.6	1.6
17	21.1	4.1	20.0	3.0
18	17.4	-.6	16.2	-1.7
11	14.4	3.4	13.2	2.2
13	11.9	-1.1	10.7	-2.3

5. Concluding Remarks

The purpose of this paper has been to give two statistically justifiable and computationally simple methods, the moment and the regression, as an alternative to the maximum likelihood approach which suffers from two deficiencies: the standard regularity conditions for the joint asymptotic normality of the maximum likelihood estimators are not satisfied and the estimating equations are either implicit or require the use of charts.

Listings of short computer programs that calculate either the moment or regression estimates, standard deviations, and the adequacy of fit statistics are available from the author. The moment and regression estimates and their standard deviations are also readily computed by hand.

The methods discussed here should also be useful in other cases where the data is multinomial and the standard maximum likelihood regularity conditions are not satisfied.

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 1704	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER (9) Technical
4. TITLE (and Subtitle) (6) SIMPLIFIED POINT AND INTERVAL ESTIMATION FOR REMOVAL TRAPPING,		5. TYPE OF REPORT & PERIOD COVERED Summary Report - no specific reporting period
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) (10) Andrew P. Soms		8. CONTRACT OR GRANT NUMBER(s) (15) DAAG29-75-C-0024
9. PERFORMING ORGANIZATION NAME AND ADDRESS Mathematics Research Center, University of Wisconsin 610 Walnut Street Madison, Wisconsin 53706		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office P.O. Box 12211 Research Triangle Park, North Carolina 27709		12. REPORT DATE (11) December 1976
		13. NUMBER OF PAGES 19
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) (12) 22p.		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

(14) MRC-TSR-1704

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Removal trapping, moment estimator, regression estimator, asymptotic confidence intervals

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Two methods, the generalized moment and the regression, both based on the limiting distribution of the multinomial, are given for estimating the parameters in the removal trapping method of estimating animal and insect populations. Some finite sample size results are provided indicating the speed of convergence to the limiting distribution. Numerical examples are also discussed.

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